How to image an electron lattice

• Composite fermions waltz to the tune of a Wigner crystal •

Y. Liu, H. Deng, M. Shayegan, L.N. Pfeiffer, K.W. West, K.W. Baldwin,
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Recommended with a commentary by Carlo Beenakker, Leiden University

The notion of a crystal of electrons, known as a Wigner crystal, actually first made its appearance in a 1932 paper by Ralph Kronig, in an unsuccessful attempt to explain superconductivity of metals at low temperatures in terms of the rigid motion of the electron lattice. Several years later Wigner evaluated the competition between the Coulomb repulsion that favors a crystal structure and the kinetic (Fermi) energy of a degenerate electron gas that would cause the crystal to melt. Since the potential energy scales as the inverse $1/r_0$ of the mean electron separation, while the kinetic energy scales as $k_F^2 \propto 1/r_0^2$, low electron densities $\rho = 1/r_0^d$ are needed to stabilize a $d$-dimensional crystal. Wigner concluded that “the electron gas differs rather curiously from an ordinary gas in so far as in the latter the potential between the atoms can be neglected if the density is small. In the electron gas, the greater the density, the more perfect the degeneracy and the behaviour becomes ideal for high densities.”

To demonstrate the appearance of a Wigner crystal in an unambiguous way, one would like to image the lattice structure. That was not possible in Wigner’s time, but also in the subsequent decades all evidence for crystallization of a degenerate electron gas had remained indirect (such as evidence in the conductivity for pinning of the crystal by defects, and evidence in the sound wave absorption for vibrational eigenmodes). The Princeton group now reports on an ingenious method to image the formation of a Wigner crystal in a two-dimensional electron gas, by using quasiparticles in a parallel layer as a probe.

The 2D electron gas is in a strong perpendicular magnetic field $B$, which forces the electrons to have the same quantized kinetic energy $\hbar eB/2m$ of the first Landau level, irrespective of their separation $r_0 = \sqrt{1/\rho}$ — as long as $r_0$ is larger than the magnetic length $l_m = \sqrt{\hbar/eB}$. The ratio $l_m/r_0 = (\nu/2\pi)^{1/2}$ is set by the filling factor $\nu$ of the Landau level, which according to calculations should be smaller than $1/7$ to allow for a Wigner crystal. (At higher filling factors Laughlin’s incompressible liquid is more stable than the crystal.)

Earlier this year, in PRL 113, 076804 (2014), a group from Stuttgart reported on an experiment in which a bilayer of two parallel 2D electron
gases was used as a capacitor to search for thermodynamic signatures of the formation of a Wigner crystal. The Princeton group studies a similar GaAs/AlGaAs bilayer, adding one key innovation: The density difference in the two layers is adjusted such that the bottom layer has the filling fraction $\nu \ll 1$ required for crystallization, while the top layer has filling fraction $\nu'$ close to 1/2.

Such a half-filled Landau level has the remarkable property that its quasiparticle excitations feel an effective field $B^* = B - 2\rho' h/e = B(1 - 2\nu')$ close to zero. These are composite fermions, consisting of electrons bound to $2h/e$ flux tubes that cancel the applied magnetic field. The cyclotron orbit radius $R^* = mv/eB^*$ of the composite fermions in the top layer can be made large enough that it becomes commensurable with the lattice constant $r_0$ of the periodic potential produced by the Wigner crystal in the bottom layer. Because $r_0$ is of order 100 nm, while the separation of the layers is only 10 nm, there is sufficient spatial resolution.

For a triangular lattice (the expected lattice structure of a 2D Wigner crystal) the resistance of the top layer should show a peak at values of $B^*$ that satisfy the commensurability condition

$$2R^*/r_0 = p(n - 1/4), \quad n = 1, 2, 3, \ldots, \quad p \in \{\frac{1}{2}, \frac{1}{2}\sqrt{3}\}.$$ (At these magnetic fields the guiding center drift of a cyclotron orbit vanishes, see arXiv:1208.4480.) The peak magnetic field values, measured from the half-filled Landau level, should therefore have ratios $1 : 0.577 : 0.429 : 0.273 \cdots$. This is not quite the sequence of ratios calculated in the paper, and not quite the sequence of ratios measured. It might be worthwhile to do a semiclassical trajectory simulation in a triangular lattice potential to be certain about the expected location of the resonances.

There are other details in the experiment that may require further clarification (the measured lattice constant is larger than expected from the electron density in the bottom layer), but this is clearly an experimental technique with great potential. Effectively (in a mean-field description), the Princeton group has succeeded in switching a magnetic field from 10 Tesla to near zero over a distance of 10 nanometer. The authors imagine using this technique to image a variety of spin- and charged-ordered phases that may compete with the Laughlin liquid.